Big Book of Helpful Hints for Yr 11 General Maths

Finding missing values in a ratio

Example : find the missing value for the equivalent ratios 3 : 7 =   : 28

Let x stand for the unknown number and write the ratios as fractions $\frac{3}{7} = \frac{x}{28}$

menu → 3:Algebra → 1:solve $\left(\frac{3}{7}=\frac{x}{28}, x\right)$ x = 12

1E Dividing quantities in given ratios

Example : calculate the number of students in each class if 60 students are divided into classes in the ratio of

	a.) 3 : 1	b.) 1 : 2 : 7
Add up the total number of parts.	3 + 1 = 4	1 + 2 + 7 = 10
Divide the number of students (60) by the number of parts to give the number of students per part.	$\frac{60}{4}$ = 15	$\frac{60}{10}$ = 6
Groups will have :	1 × 15 = 15 students	1 × 6 = 6 students
	3 × 15 = 45 students	2 × 6 = 12 students
		7 × 6 = 42 students

1F Unitary method

Find the value of 1 item and then multiply by any number you like.

Example : 24 golf balls cost $86.40, how much do 7 golf balls cost?

24 golf balls = $86.40

$\frac{24}{24}$ golf balls = $\frac{\$86.40}{24}$ = $3.60

1 golf ball = $3.60

7 golf balls = 7 × $3.60 = $25.20

2 : INVESTIGATING AND COMPARING DATA DISTRIBUTIONS

2A Classifying and displaying categorical data

Categorical data

is used to group things into categories. Like sorting things into bins.

→ Nominal data

such as Male, M, or Female, F, are simply names. Labels on the bins.

Yellow, Red, Blue. Good, Bad, Ugly.

→ Ordinal data

such as ‘small’, ‘medium’ and ‘large’. Labels on the bins for ordering things.

essential, important, optional

Numerical data

can be used for arithmetic, like adding or averaging.

→ Discrete data

for things that come in countable lumps.

How many brothers do you have?

→ Continuous data

can take any numerical value within a specified range.

How much toothpaste did you squeeze out of the tube?

Probably between 0.0cm to 0.6cm. Most of us at 0.4cm?

The frequency table

lists the values in a data set, and how often (frequently) each value occurs.

Frequency can be recorded as a:

- frequency: the number of times a value occurs
- percentage frequency: the percentage of times a value occurs,

  where: percentage frequency = $\frac{\text{count}}{\text{total}} \times 100$

- frequency distribution: a listing of the values a variable takes, along with how frequently each of these values occurs.

Example : thirty children chose a sandwich, a salad or a pie for lunch, as follows:

sandwich × 7

salad × 10

pie × 13

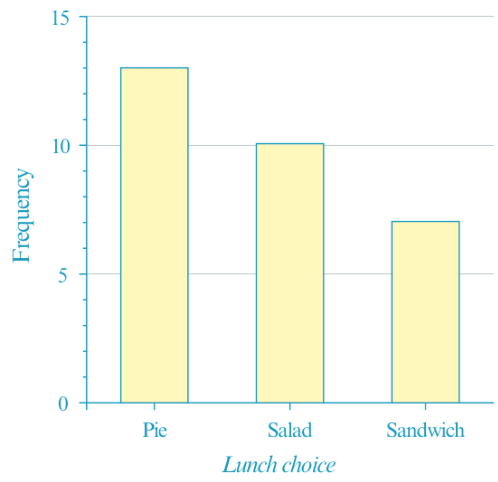
Lunch choice	Frequency	
	Number	%
Sandwich	7	23.3
Salad	10	33.3
Pie	13	43.3
Total	30	99.9

Bar charts

are for categorical data

In a bar chart:

- frequency or percentage frequency is shown on the vertical axis
- the variable being displayed is plotted on the horizontal axis
- the height of the bar (column) gives the frequency (or percentage)
- the bars are drawn with gaps to indicate that each value is a separate category
- there is one bar for each category.



Bar chart

Percentage bar chart

The mode or modal category

is the most frequently occurring category.

is important for answering questions like ‘When is a supermarket in peak demand?’

2B Interpreting and describing frequency tables and bar charts

As part of this topic, you will be expected to complete a statistical investigation.

Some guidelines for describing the distribution of a categorical variable and communicating your findings

- ■ Briefly summarise the context in which the data were collected including the number of people (or things) involved in the study.
- ■ If there is a clear modal category, make sure that it is mentioned.
- ■ Include relevant counts or percentages in the report.
- ■ If there are a lot of categories (more than 3), it is not necessary to mention every category.
- ■ Either counts or percentages can be used to describe the distribution.

Describing the distribution of a categorical variable from a frequency table.

A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch, and their responses were collected and summarised in the frequency table opposite.

Use the frequency table to report on the relative popularity of the three lunch choices, quoting appropriate frequencies to support your conclusions.

Lunch choice	Frequency
Sandwich	7
Salad	10
Pie	13
Total	30

Report

A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch.

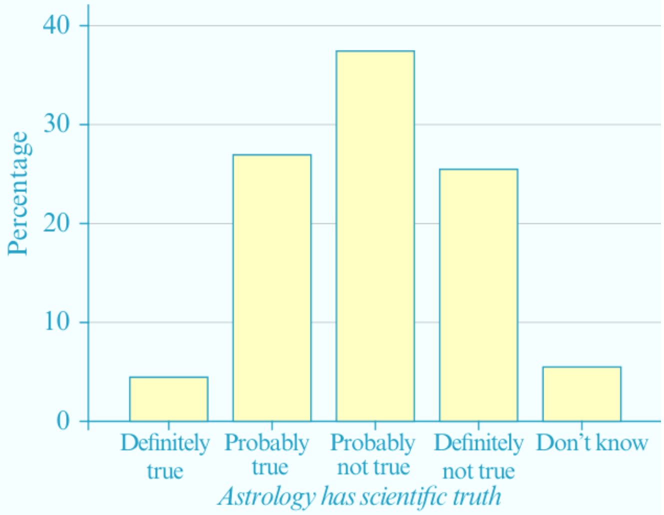
The most popular lunch choice was a pie, chosen by 13 of the children. Ten children chose a salad. The least popular option was a sandwich, chosen by only 7 of the children.

Describing the distribution from a frequency table and bar chart.

A sample of 200 people were asked to comment on the statement ‘Astrology has scientific truth’ by selecting one of the options ‘definitely true’, ‘probably true’, ‘probably not true’, definitely not true’ or ‘don’t know’.

The data are summarised in the following frequency table and bar chart (in a definite order because the data are ordinal).

Astrology has scientific truth	Frequency	
	Number	%
Definitely true	9	4.5
Probably true	54	27.0
Probably not true	75	37.5
Definitely not true	51	25.5
Don’t know	11	5.5
Total	200	100.0



Write a report using the frequency table and bar chart.

Report

A sample of two hundred people were asked to respond to the statement ‘Astrology has scientific truth’.

The majority of respondents did not agree, with 37.5% responding that they believed that this statement was probably not true, and another 25.5% declaring that the statement was definitely not true. Over one quarter (27%) of the respondents thought that the statement was probably true, while only 4.5% thought that the statement was definitely true.

2C Displaying and describing numerical data

Discrete data, small number of values

use each value as a category, as shown below

The number of brothers and sisters (siblings) reported by each of the 30 students in Year 11 are as follows:

2 3 4 0 3 2 3 0 4 1 0 0 1 2 3 0 2 1 1 4 5 3 2 5 6 1 1 1 0 2

Construct a table for these data showing both frequency and percentage frequency.

1 Find the maximum and the minimum values in the data set. Here the minimum is 0 and the maximum is 6.

2 Construct a table as shown, including all the values between the minimum and the maximum.

3 Count the number of 0s, 1s, 2s, etc. in the data set. For example, there are seven 1s.

Record these values in the number column.

4 Add the frequencies to find the total.

5 Convert the frequencies to percentages, and record in the per cent (%) column.

Number of siblings	Frequency	
	Number	%
0	6	20.0
1	7	23.3
2	6	20.0
3	5	16.7
4	3	10.0
5	2	6.7
6	1	3.3
Total	30	100.0

Grouping data

when the variable can take on a large range of values (e.g. age from 0 to 100 years) or when the variable is continuous (e.g. response times measured in seconds to two decimal places), we group the data into a small number of convenient intervals.

These grouping intervals should be chosen according to the following principles:

■ Every data value should be in an interval.

■ The intervals should not overlap.

■ There should be no gaps between the intervals.

The choice of intervals can vary but there are some guidelines.

■ A division which results in about 5 to 15 groups is preferred.

■ Choose an interval width that is easy for the reader to interpret, such as 10 units, 100 units or 1000 units (depending on the data).

■ By convention, the beginning of the interval is given the appropriate exact value, rather than the end. As a result, intervals of 0–49, 50–99, 100–149 would be preferred over the intervals 1–50, 51–100, 101–150 etc.

Grouped discrete data

Constructing a grouped frequency table for a discrete numerical variable

A group of 20 people were asked to record how many cups of coffee they drank in a particular week, with the following results:

2 0 9 10 23 25 0 0 34 32 5 0 17 14 3 6 0 33 23 0

Construct a grouped frequency table of these data showing both frequency (count) and percentage frequency.

1 The minimum number of cups of coffee drunk is 0 and the maximum is 34. Intervals beginning at 0 and ending at 34 would ensure that all the data are included. Interval width of 5 will mean that there are 7 intervals.

Note that the endpoints are within the interval, so that the interval 0 – 4 includes 5 values: 0, 1, 2, 3 and 4.

2 Set up the table as shown.

3 Count the data values in each interval to complete the number column.

Cups of coffee	Frequency	
	Number	%
0 – 4	8	40
5 – 9	3	15
10 – 14	2	10
15 – 19	1	5
20 – 24	2	10
25 – 29	1	5
30 – 34	3	15
Total	20	100

4 Convert the frequencies into percentages and record in the per cent (%) column.

For example, for the interval 5–9: % frequency = $\frac{3}{20} \times 100 = 15\%$.

5 Total the percentages and record.

Grouped continuous data

Constructing a frequency table for a continuous numerical variable

The following are the heights of the 41 players in a basketball club, in centimetres.

178.1 185.6 173.3 193.4 183.1 184.6 202.4 170.9 183.3 180.3 185.8 189.1 178.6 194.7

185.3 191.1 189.7 191.1 180.4 180.0 193.8 196.3 189.6 183.9 177.7 178.9 193.0 188.3

189.5 182.0 183.6 184.5 188.7 192.4 203.7 180.1 170.5 179.3 184.1 183.8 174.7

Construct a frequency table and a percentage frequency table for these data.

1 Find the minimum and maximum heights, which are 170.5 cm and 203.7 cm.

A minimum value of 170 and a maximum of 204.9 will ensure that all the data are included.

2 Interval width of 5 cm will mean that there are 7 intervals from 170 to 204.9, which is within the guidelines of 5–15 intervals.

3 Set up the table as shown. All values of the variable that are from 170 to 174.9 have been included in the first interval.

The second interval includes values from 175 to 179.9, and so on for the rest of the table.

Height (cm)	Frequency	
	Number	%
170−174.9	4	9.8
175−179.9	5	12.2
180−184.9	13	31.7
185−189.9	9	22.0
190−194.9	7	17.1
195−199.9	1	2.4
200−204.9	2	4.9
Total	41	100.1

4 The number of data values in each interval is then counted to complete the number column of the table.

5 Convert the frequencies into percentages and record in the per cent (%) column.

For example, for the interval 175.0–179.9: % frequency = $\frac{5}{41} \times 100 = 12.2\%$.

6 Total the percentages and record.

The interval that has the highest frequency is called the modal interval. In the example above, the modal interval is 180.0–184.9, as 13 players (31.7%) have heights that fall into this interval.

Histograms

are for numerical variables.

■ frequency (number or percentage) is shown on the vertical axis

■ the values of the variable being displayed are plotted on the horizontal axis

■ each column corresponds to a data value, or a data interval if the data is grouped; alternatively, for ungrouped discrete data, the actual data value is located at the middle of the column

■ the height of the column gives the frequency (number or percentage).

Constructing a histogram for ungrouped discrete data

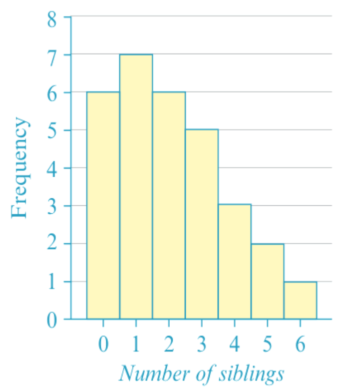
Construct a histogram for the data in the frequency table.

Number of Siblings	Frequency
0	6
1	7
2	6
3	5
4	3
5	2
6	1
Total	30

1 Label the horizontal axis with the variable name Number of siblings.

Mark in the scale in units that include all possible values.

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 7. Up to 8 would be appropriate. Mark the scale in units.



3 For each value of the variable, draw in a column. The data is discrete, so make the width of each column 1, starting and ending halfway between data values.

For example, the column representing 2 siblings starts at 1.5 and ends at 2.5.

The height of each column is equal to the frequency.

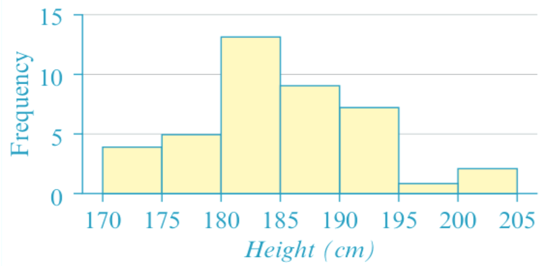
Constructing a histogram for continuous data

Construct a histogram for the data in the frequency table.

Height (cm)	Frequency
170−174.9	4
175−179.9	5
180−184.9	13
185−189.9	9
190−194.9	7
195−199.9	1
200−204.9	2
Total	41

1 Label the horizontal axis with the variable name Height (cm). Mark in the scale using the beginning of each interval as the scale points; that is, 170, 175, …

2 Label the vertical axis ‘Frequency’. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in units.



3 For each interval, draw in a column. Each column starts at the beginning of the interval and finishes at the beginning of the next interval. Make the height of each column equal to the frequency.

2D Characteristics of distributions, dot plots and stem plots

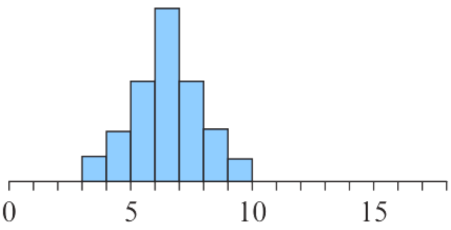
Characteristics of a distribution

are shape, location (also referred to as the ‘centre’) and spread.

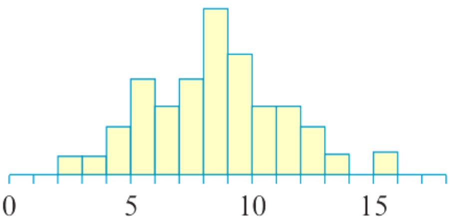
Shape of a distribution, symmetric or skewed

Symmetric distribution

forms a mirror image of itself when folded in the ‘middle’ along a vertical axis.



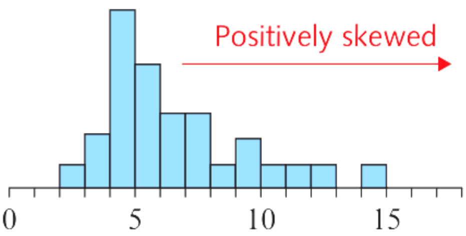
Histogram A
is exactly symmetric.



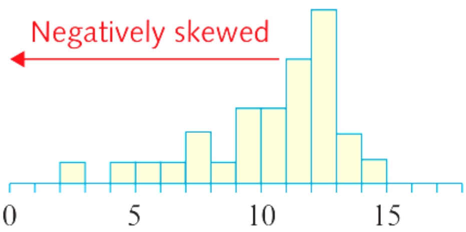
Histogram B
is more realistic,
shows enough symmetry to classify
this histogram as symmetric.

Positive and negative skew

- Positively skewed because of the many values towards the positive end of the distribution.
- Negatively skewed because of the many values towards the negative end of the distribution.



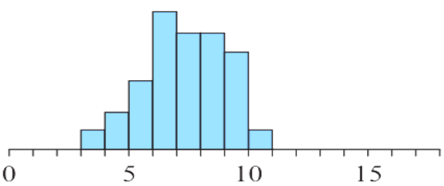
Histogram C



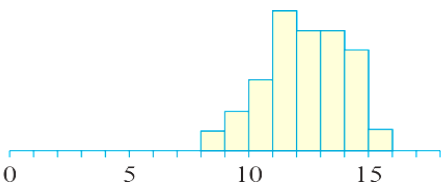
Histogram D

Comparing centre or location

Two distributions differ in centre if the values of the data in one distribution are generally larger than the values of the data in the other distribution.



Histogram E



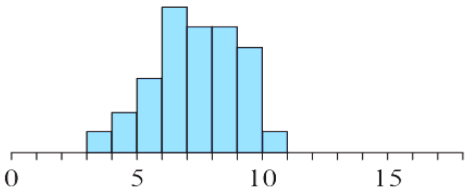
Histogram F

Histogram F is identical in shape and width to Histogram E but is moved several units to the right, indicating that these distributions differ in location.

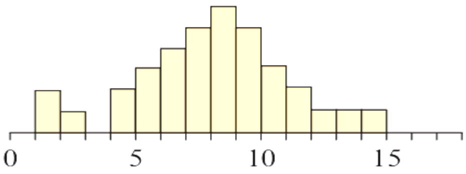
Comparing spread

Two distributions are said to differ in spread if the values of the data in one distribution tend to be more variable (spread out) than the values of the data in the other distribution.

Histograms G and H are both are centred at about the same place, but Histogram H is more spread out.



Histogram G



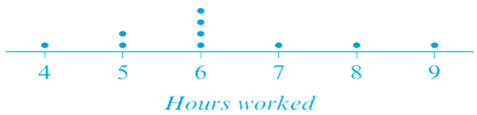
Histogram H

Dot plots

display fairly small data sets where the data takes a limited number of values.

The number of hours worked by each of 10 students in their part-time jobs is as follows:

6, 9, 5, 8, 6, 4, 6, 7, 6, 5



Construct a dot plot of these data.

Stem plots

The following is a set of marks obtained by a group of students on a test:

15, 2, 24, 30, 25, 19, 24, 33, 18, 60, 42, 37, 28, 28, 17, 19, 52, 55, 27, 5, 7, 19, 45, 19, 25

Display the data in the form of an ordered stem plot.

0	2 5 7
1	5 9 8 7 9 9 9
2	4 5 4 8 8 7 5
3	0 3 7
4	2 5
5	2 5
6	0

unordered

Marks key: 1 | 5 = 15 marks

0	2 5 7
1	5 7 8 9 9 9 9
2	4 4 5 5 7 8 8
3	0 3 7
4	2 5
5	2 5
6	0

ordered, complete with title and key

Stem is the leading digit or digits. Leaf as the final digit.

Always include a key.

2E Measures of centre, mean and median

Mean, $\bar{x}$

means average

$\bar{x}$ means add up all the (Σ) numbers (x) and then divide by how many there are (n)

$\bar{x} = \frac{\sum x}{n}$ mean of 4, 5 and 6 is $\frac{4+5+6}{3} = 5$

Median

is the middle number, when the numbers are arranged in order of size

median = 6

odd number of data 2 3 4 5 5 6 7 7 8 8 11

median value is data point at $\frac{\text{number of data} + 1}{2}$

median = $\frac{6+7}{2}$ = 6.5

even number of data 2 3 4 5 5 6 7 7 8 8 11 11

median value is average of data points at $\frac{\text{number of data}}{2}$ and $\frac{\text{number of data}}{2} + 1$

(Important : can you find the median in a stem-and-leaf plot?)

2F Measures of spread

Range (R)

is the difference between the largest value and smallest value

R = biggest number − smallest number

Interquartile range (IQR)

IQR = upper quartile − lower quartile = $Q_3 - Q_1$

- Odd number

[1 2 2 3] 5 [6 6 7 9]

$Q_1(2)$ $Q_2(5)$ $Q_3(6.5)$

IQR (4.5)

- Even number

[2 3 3 4 7] [8 8 9 9 9]

$Q_1(3)$ $Q_2(7.5)$ $Q_3(9)$

IQR (6)

■ divide the data into two equal-sized groups, and if n is odd, omit the median from both groups.

■ Q_1 is the median of the lower half of the data,

Q_3 is the median of the upper half of the data.

IQR = $Q_3 - Q_1$.

IQR gives the spread of the middle 50% of data values. (It′s the length of the box.)

Standard deviation, s

$$s = \sqrt{\left(\frac{\sum (x-\bar{x})^2}{n-1}\right)}$$

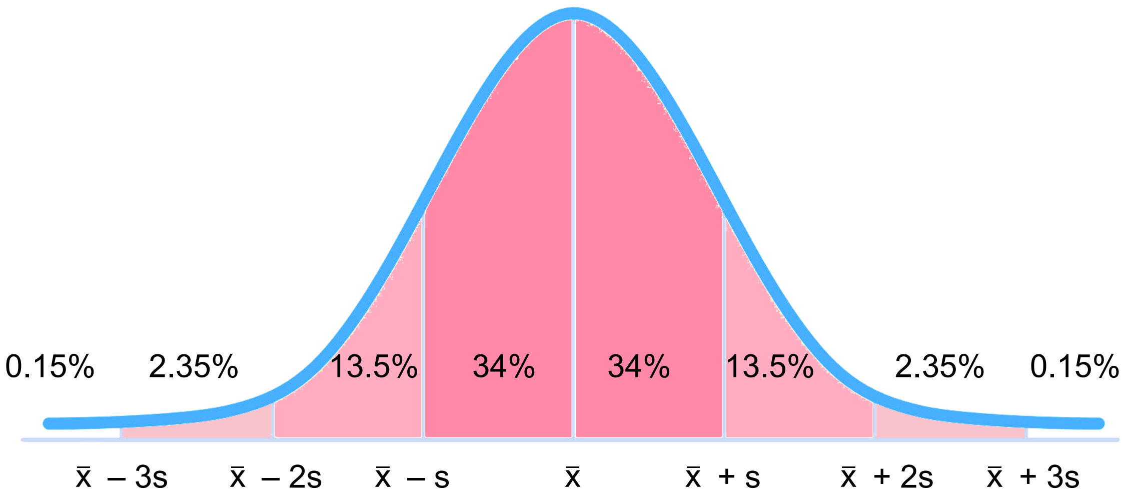
where n is the number of data values (sample size) and $\bar{x}$ is the mean.

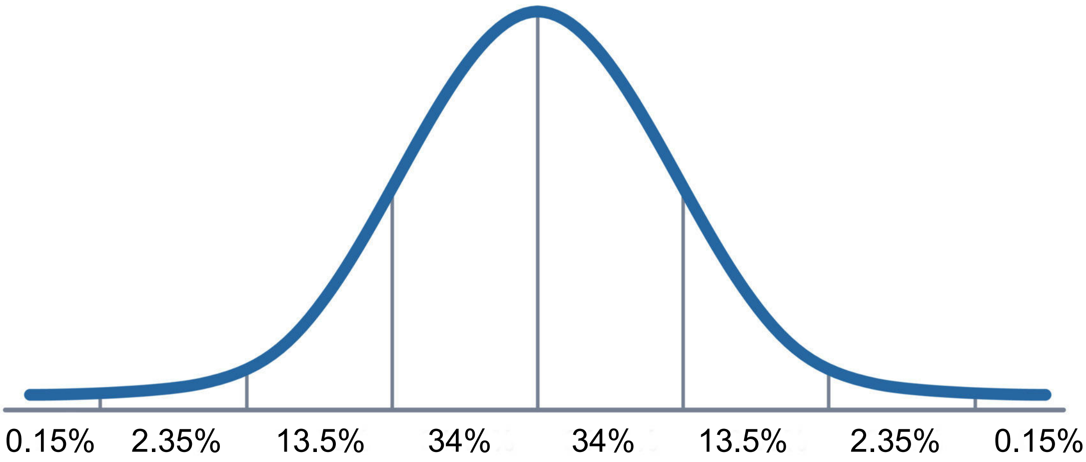
2G Percentages of data lying within multiple standard deviations of the mean

The 68 - 95 - 99.7% rule

For any data distribution which is approximately symmetric and bell shaped, approximately:

68% lie within $\bar{x} \pm s$ 95% lie within $\bar{x} \pm 2s$ 99.7% lie within $\bar{x} \pm 3s$





Example : percentages of data lying within 1, 2 or 3 standard deviations of the mean

The distribution of the examination scores for a very large statewide examination is approximately symmetric and bell shaped, with a mean of 65 and a standard deviation of 10.

a.) Approximately what percentage of students scored between 55 and 75?

b.) Approximately what percentage of students scored between 45 and 85?

c.) Approximately what percentage of students scored between 35 and 95?

a.) A score of 55 is 1SD below the mean of 65 and a score of 75 is 1SD above the mean.	Approximately 68% of the scores are between 55 and 75.
b.) A score of 45 is 2SD below the mean of 65 and a score of 85 is 2SD above the mean.	Approximately 95% of the scores are between 45 and 85.
c.) A score of 35 is 3SD below the mean of 65 and a score of 95 is 3SD above the mean.	Approximately 99.7% of the scores are between 35 and 95.

Example : finding the interval for a given percentage

The distribution of the diameter of bolts produced in a factory is approximately symmetric and bell shaped, with a mean of 5 mm and with a standard deviation of 0.01mm.

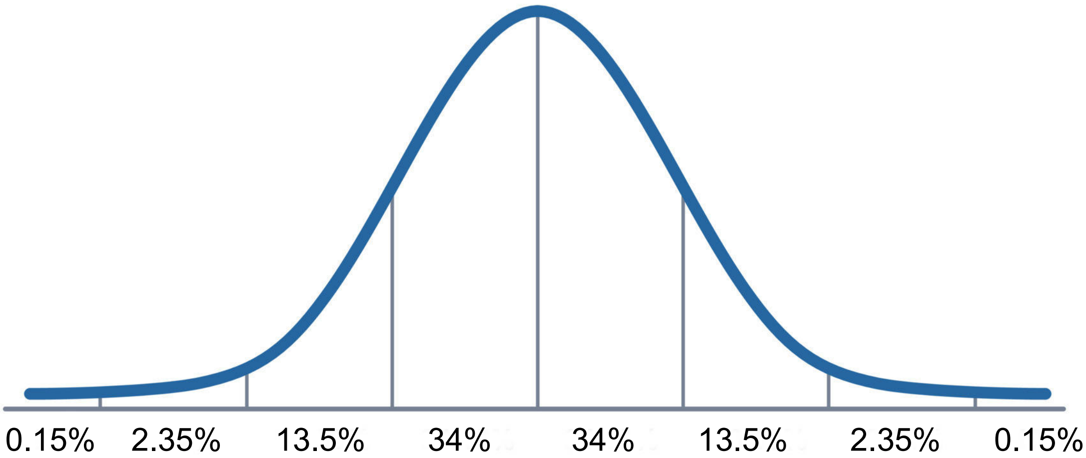
a.) If approximately 68% of the bolts measure between a and b, what are possible values for a and b?

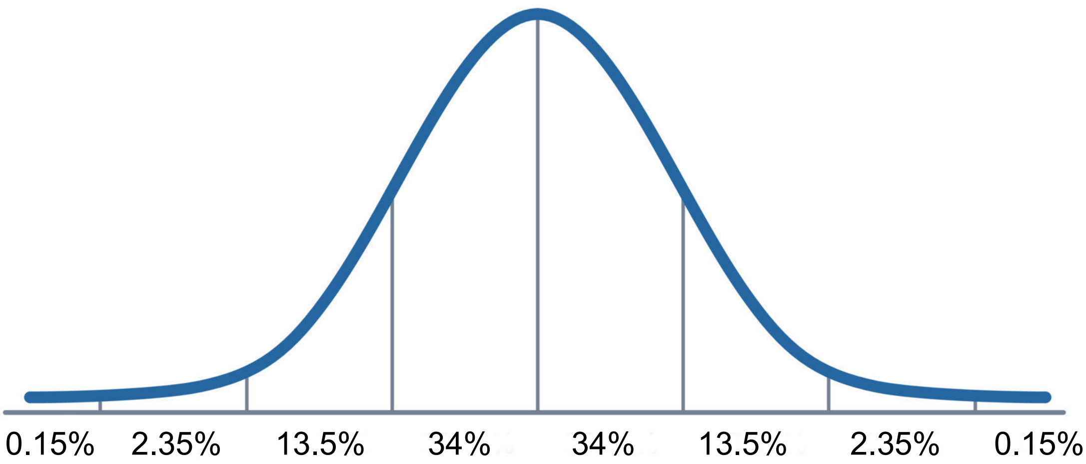
b.) If approximately 95% of the bolts measure between c and d, what are possible values for c and d?

c.) If approximately 99.7% of the bolts measure between e and f, what are possible values for e and f?

a.) The interval which contains 68% of the bolts is 1SD either side of the mean.	a = 5 - 0.01 = 4.99 mm b = 5 + 0.01 = 5.01 mm
b.) The interval which contains 95% of the bolts is 2SD either side of the mean.	c = 5 - 2 × 0.01 = 4.98 mm d = 5 + 2 × 0.01 = 5.02 mm
c.) The interval which contains 99.7% of the bolts is 3SD either side of the mean	e = 5 - 3 × 0.01 = 4.97 mm f = 5 + 3 × 0.01 = 5.03 mm

Bell curve blanks





2H Boxplots, simple and with outliers

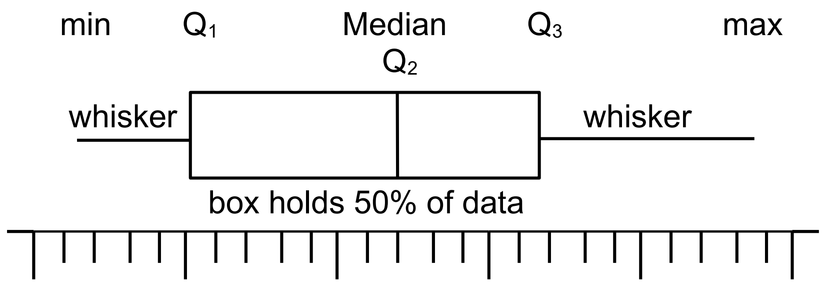
Five-figure summary

of data arranged in ascending order.

- Minimum value (min)
- Lower quartile (Q_1) : median of bottom half. 25% of data is below this number.
- Median (Q_2) : the median value. 50% of data is above this number, 50% below
- Upper quartile (Q_3) : the median of top half. 75% of the data is below this number.
- Maximum value (max)

Simple boxplot

is the five-number summary in pictorial (graphical) form



Example : the following are the monthly rainfall figures for a year in Melbourne.

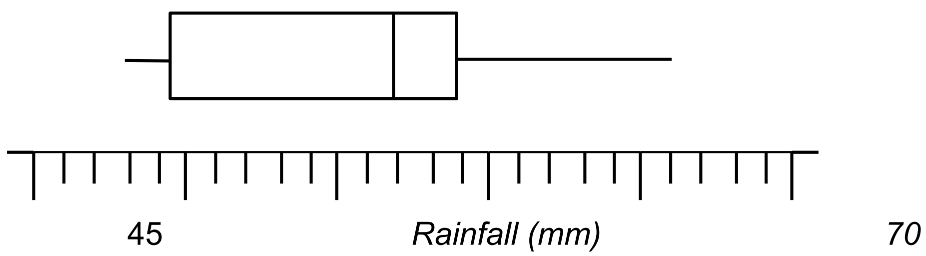
Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Construct a boxplot to display this data.

48, 49, 49, 50, 52, 57, 57, 58, 59, 59, 60, 67

[48, 49, 49, 50, 52, 57] [57, 58, 59, 59, 60, 67]

Min = 48 Q1 = $\frac{49+50}{2}$ = 49.5 M = $\frac{57+57}{2}$ = 57 Q3= $\frac{59+59}{2}$ = 59 Max = 67



Outliers, data beyond the fences

Outlier : any data point that is

smaller than $Q_1 - 1.5 \times IQR$ (lower fence)

or

bigger than $Q_3 + 1.5 \times IQR$ (upper fence)

Shown with a dot or a cross on a boxplot.

Example : the number of hours that 33 students spent on a school project is shown below.

2 3 4 9 9 13 19 24 27 35 36 37 40 48 56 59 71

76 86 90 92 97 102 102 108 111 146 147 147 166 181 226 264

Construct a boxplot and identify possible outliers.

Odd number of data points. Median value at point $\frac{33+1}{2}$ which is 71.

Divide data into two groups. 71 omitted from both groups.

Q_1

[2 3 4 9 9 13 19 24 27 35 36 37 40 48 56 59]

16 = even number of data points. Q_1 is average of values at $\frac{16}{2}$ and $\frac{16}{2}+1$

$Q_1 = \frac{24+27}{2} = 25.5$

Q_3

[76 86 90 92 97 102 102 108 111 146 147 147 166 181 226 264]

Q_3 is average of values at $\frac{16}{2}$ and $\frac{16}{2}+1$

$Q_3 = \frac{108+111}{2} = 109.5$

$IQR = Q_3 - Q_1 = 109.5 - 25.5 = 84$

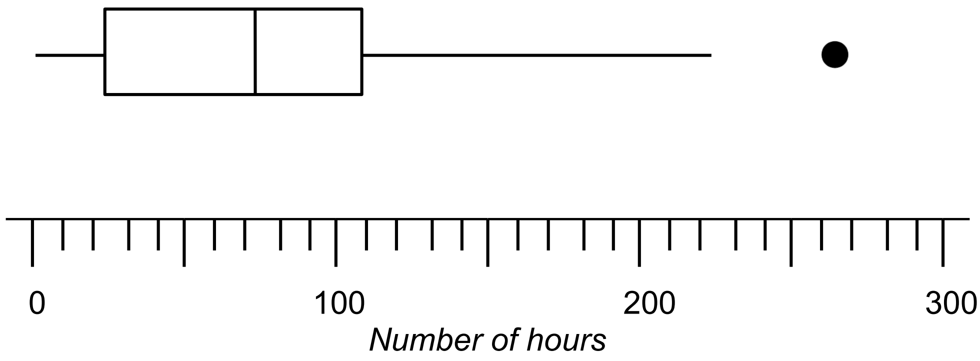
Lower fence = $Q_1 - 1.5 \times IQR$

Upper fence = $Q_3 + 1.5 \times IQR$

$= 25.5 - 1.5 \times 84$ $= 109.5 + 1.5 \times 84$

$= -100.5$ $= 235.5$

264 is above the upper fence, so it is an outlier and will be drawn with a dot.
The whisker will extend to 226, which is the largest value that is not an outlier.



There is one outlier, the student who spent 264 hours.

2I Comparing the distribution of a numerical variable across groups

Comparing distributions using back-to-back stem plots

This back-to-back stem plot displays the distributions of life expectancies for males (in years) in several countries in the years 1970 and 2010.

In this situation, Male life expectancy is the numerical variable. Year, which takes
the values 1970 and 2010, is the categorical variable.

Use the back-to-back stem plot and the summary statistics provided to compare these distributions in terms of centre and spread, and draw an appropriate conclusion.

Male life expectectancy (in years)

1970 2010

8 | 3 = 38 years 5 | 8 = 58 years

1970		2010
8	3	
	4	
9 4 2	5	8
9 9 9 8 7 7 1 0	6	9
4	7	1 2 4 4 6
	8	0 0 0

M = 67 years M = 76 years

IQR = 12.5 years IQR = 8 years

1 Centre: Write a sentence using the medians to compare centres.

The median life expectancy of males in 2010 (M = 76 years) was higher than in 1970 (M = 67 years).

2 Spread: Write a sentence using the IQRs to compare spreads.

The spread of life expectancies of males in 2010 (IQR = 8 years) was lower than the spread in 1970 (IQR = 12.5 years).

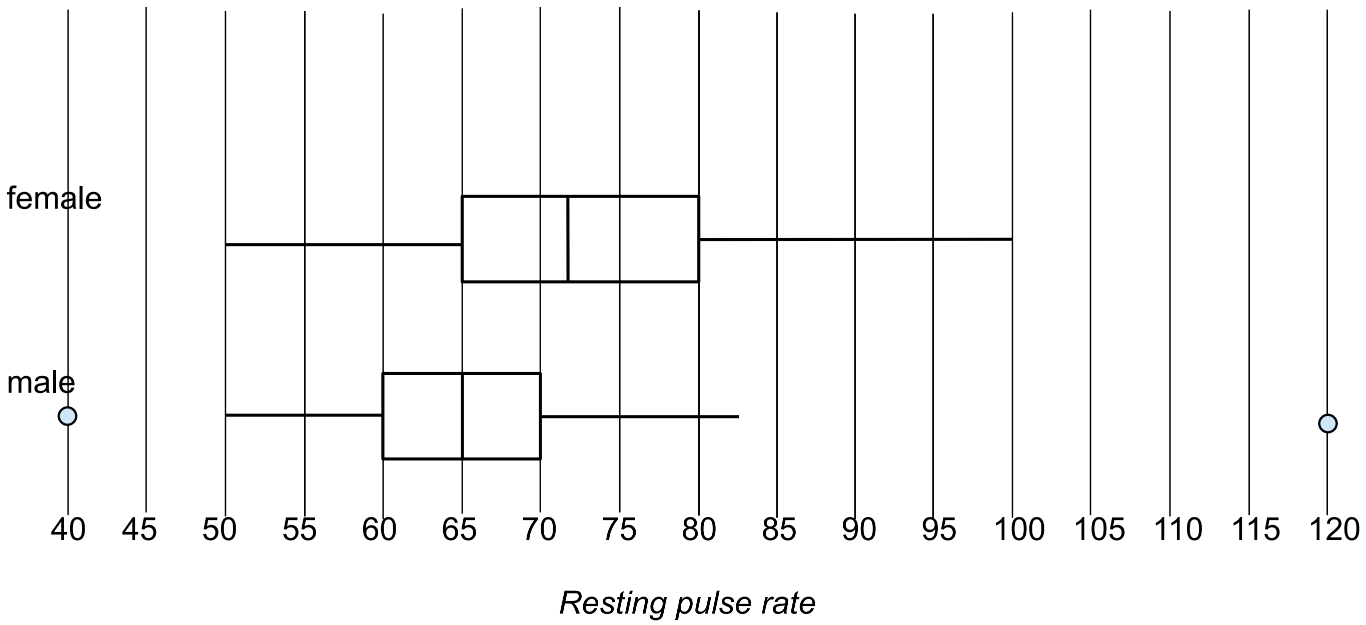
3 Conclusion: Use the above observations to add a general conclusion.

In conclusion, the median life expectancy for men in these countries has increased over the last 40 years, and the variability in male life expectancy has decreased over this time interval.

Comparing distributions using parallel boxplots

The following parallel boxplots display the distribution of pulse rates (in beats/minute) for a group of female students and a group of male students.

Use the information in the boxplots to write a report comparing these distributions in terms of centre, spread and outliers in the context of the data.



1 Centre: Determine values of the medians from the plot (the vertical lines in the boxes), and write a sentence comparing these values.	The median pulse rate for females (M = 72 beats/minute) is higher than that for males (M = 65 beats/minute).
2 Spread: Determine the spread of the two distributions using IQRs (the widths of the boxes), and write a sentence comparing these values.	The spread of pulse rates for females (IQR = 15) is higher than for males (IQR = 10).
3 Outliers: Locate any outliers and write a sentence describing these.	There are no female pulse rate outliers. The males with pulse rates of 40 and 120 were outliers.
4 Conclusion: Add a general conclusion based on these comparisons.	In conclusion, the median pulse rate for females was higher than for males, and female pulse rates were generally more variable than male pulse rates.

3 : SEQUENCES AND FINANCE

The textbook bangs on and on. These notes do not. But they contain what you need.

3A Number patterns

Sequence

is a list of numbers. Each number is called a term, t, and its place on the list is shown with a subscribt.

Arithmetic sequence has a common difference, D. 3, 7, 10, 13, 16 $t_2 = 7$

Geometric sequence has a common ratio, R. 3, 9, 27, 81, 243 $t_3 = 27$

Recursion

is the process of generating a sequence of terms from a given starting point and a rule.

Recurrence relation is the tool we use to do it.

3B Writing recurrence relations in symbolic form

Recurrence relation

is the maths name for the rule that is used to generate a sequence.

Has two parts:

- a starting point: the value of the term at the start of the sequence
- a rule, that can be used to generate successive terms in the sequence.

Example

Starting value (n = 0)	Rule for generating the next value	Recurrence relation is the combination of the two
$t_0 = 3$	$t_{n+1} = t_n + 6$	$t_0 = 3$, $t_{n+1} = t_n + 6$

4D Matrix multiplication

number of **columns in 1st** matrix must be **same as** number of **rows in 2nd** matrix.	$(m \times n)\ (n \times p) = (m \times p)$ ↑ ↑ must be the same
product matrix has number of **rows** from **1st** matrix number of **columns** from **2nd** matrix.	$(m \times n)\quad (n \times p) \quad = \quad (m \times p)$ this many columns this many rows

Multiply each row in A by each column in B. (Run along the row, dive into the column.)

Multiply row m by column p and the result goes in a_{mp} of the product matrix.

Multiply row 1 by column 1 and the result goes in a_{11} of the product matrix.

Multiply row 2 by column 1 the result goes in a_{21} of the product matrix.

Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ Then $AB = \begin{bmatrix} 1\times 4 + (-2)\times 1 \\ (-2)\times 4 + 0\times 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 10 \\ 9 & 8 \end{bmatrix}$ Then $AB = \begin{bmatrix} 2\times 7+3\times 9 & 2\times 10+3\times 8 \\ 5\times 7+4\times 9 & 5\times 10+4\times 8 \\ 1\times 7+6\times 9 & 1\times 10+6\times 8 \end{bmatrix} = \begin{bmatrix} 41 & 44 \\ 71 & 82 \\ 61 & 58 \end{bmatrix}$

Matrix powers

We define the various powers of matrices as:
A^2 as A × A,
A^3 as A × A × A,
A^4 as A × A × A × A and so on.

Only square matrices can be raised to a power.

Euler’s formula

For a connected planar graph:

number of vertices + number of faces = number of edges + 2

or

$v + f = e + 2$

where v = number of vertices, e = number of edges and f = number of faces.

For the graph opposite:

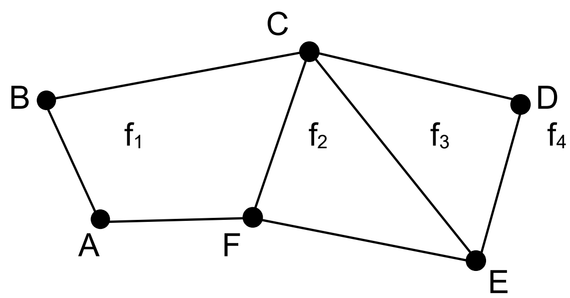
$v = 6$, $f = 4$ and $e = 8$

$v + f = e + 2$

$6 + 4 = 8 + 2$

$10 = 10$

confirming Euler’s formula.



Determining the constant of variation

Example : use the table of values to determine the constant of variation, k, and hence complete the table: $y \propto x$

x	3	5	7	
y	21		49	63

1 Rewrite the variation expression as an equation, with k as the constant of variation.

$y \propto x$

$y = kx$

2 Substitute corresponding values for x and y, and solve for k.

When $x = 3$, $y = 21$

$21 = 3k$

$\therefore k = 7$

Francis says $k = \frac{y_n}{x_n} = \frac{3}{21} = 7$

3 Substitute $k = 7$ in $y = kx$.

$y = 7x$

4 Substitute the value for x to find the corresponding y value.

When $x = 5$,

$y = 7(5)$

$\therefore y = 35$

5 Substitute the value of y to find the corresponding x value.

When $y = 63$,

$63 = 7(x)$

$\therefore x = 9$

6 Complete the table.

x	3	5	7	9
y	21	35	49	63

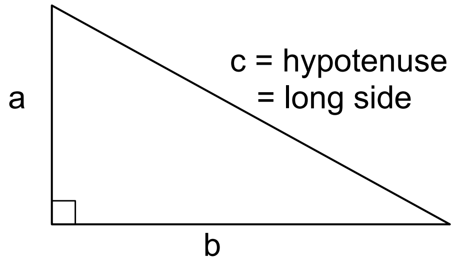
10B Pythagoras′ theorem

only applies to right angle triangles.

Basic theorem $a^2 + b^2 = c^2$

To find the hypotenuese $c = \sqrt{a^2+b^2}$

To find a short side $a = \sqrt{c^2-b^2}$



Exact value means leave your answer under the square root sign.
Do not convert it into a decimal.

Pythagoras in a box

Find hypotenuse of bottom triangle

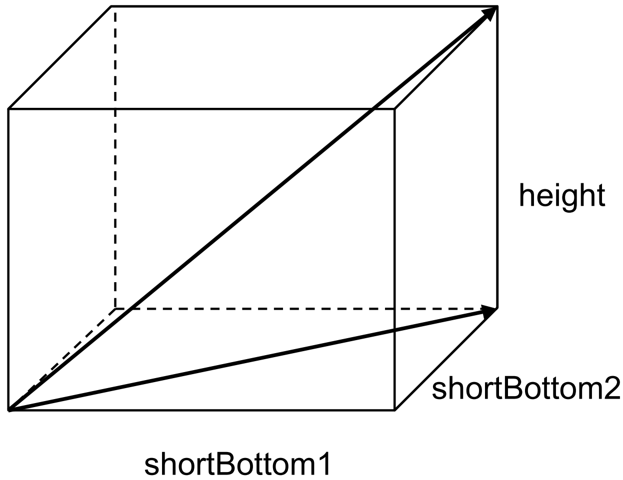
$\text{longBottom} = \sqrt{(\text{shortBottom}1)^2+(\text{shortBottom}2)^2}$

Biggest distance is hypotenuse of standing triangle

$\text{Distance} = \sqrt{(\text{longBottom})^2+(\text{height})^2}$

OR

$\text{Distance} = \sqrt{(\text{shortBottom}1)^2+(\text{shortBottom}2)^2+(\text{height})^2}$

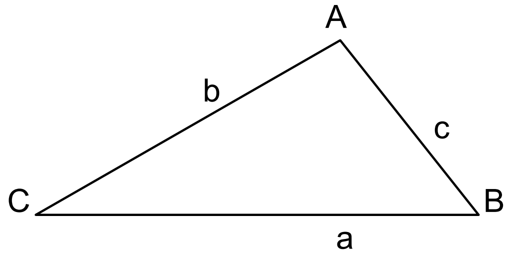


11G The sine rule

Standard triangle notation

Upper case letters, A, B, and C, for the angles at each corner.

Lower case letters for the sides, so that side a is opposite angle A, and so on.



The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Find an unknown angle, A,

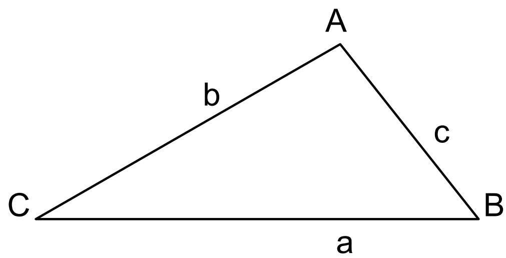
given two sides, a and b, and an opposite angle, B

Let A be the angle to find.

a is the given side opposite to angle A.

b is the side opposite to angle B.

Using b and B OR c and C makes no difference.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Rearrange the sine rule to make A the subject.

$$\frac{a \sin B}{b} = \sin A$$

$$\sin^{-1}\left(\frac{a \sin B}{b}\right) = A$$

sin B =

$$\sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}\left(\frac{\quad \times \quad}{\qquad}\right) =$$